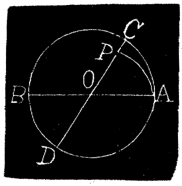
41. Proposed by F. M. SHIELDS. Coopwood, Mississippi.

A railroad turn-table 100 feet long is balanced upon a pivot in the center of a circular track 100 feet in diameter. How far does a man walk who starts at one end of the table and walks, at a uniform rate, the entire length of the table in the same time that the table makes two revolutions, if the table starts to turn at the same time the man starts to walk?

Solution by C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts.

It is the purpose of this solution to find how far the man moves in space, if he always walks on the same line CD until across.



Let $OA=a$, $OP=r$, $\angle COA=\theta$, the velocity of C around the track n times the velocity of P along CD, P being the man's position at any time. Then $n.PC=\text{arc } AC$ $=a\theta$, $\therefore\ PC=\frac{a\theta}{n}$.

$\therefore\ r=a-PC=a-\frac{a\theta}{n},\ =\frac{a(n-\theta)}{n}$. $\therefore\ r=a(n-\theta)/n$

is the equation of the man's path; also, $ds=\sqrt{(dr)^2+r^2(d\theta)^2}$, but $(dr)^2=\frac{a^2}{n^2}(d\theta)^2$. $\therefore ds=\pm\frac{a}{n}\sqrt{1+(n-\theta)^2}d\theta$. $\therefore s=-\frac{2a}{n}\int\sqrt{1+(n-\theta)^2}d\theta$, for whole length $=\frac{a}{n}(n-\theta)\sqrt{1+(n-\theta)^2}+\frac{a}{n}\log\{n-\theta+\sqrt{1+(n-\theta)^2}\}+C$, but $2na$ $=4\pi a$, $\therefore\ n=2\pi$, $\therefore\ r=a(2\pi-\theta)/2\pi$.

$$s=\frac{a}{2\pi}(2\pi-\theta)\sqrt{1+(2\pi-\theta)^2}+\frac{a}{2\pi}\log\{2\pi-\theta+\sqrt{1+(2\pi-\theta)^2}\}.$$ The limits of θ are 0 and 2π, and $a=50$. $\therefore\ s=50\sqrt{1+4\pi_2}+\frac{25}{\pi}\log\{2\pi+\sqrt{1+4\pi^2}\}$.

$\therefore\ s=338.303$ feet.

Similarly solved by *G. B. M. ZERR*.

43. Proposed by J. C. NAGLE, M. A., C. E., Professor of Civil Engineering, A. and M. College, College Station, Texas.

Show that the volume included between the surface represented by the equation $z=e^{-(x^2+y^2)}$ and the xy plane equals the square of the area of the section made by the zx plane, the limits of x and y being plus and minus infinity.

I. Solution by Professor J. SCHEFFER, A. M., Hagerstown, Maryland.

Changing to polar co-ordinates, the volume is $=\int_0^{2\pi}d\theta\int_0^{\infty}e^{-r^2}rdr=$ $\frac{1}{2}\int_0^{2\pi}d\theta=\pi$. The area is $2\int_0^{\infty}e^{-x^2}dx$. Putting in the Gamma Function

$\int_0^\infty e^{-z}z^{n-2}dz=\Gamma(n)$, $z=x^2$, $n=\frac{1}{2}$, we find $2\int_0^\infty e^{-x^2}dx=\Gamma(\frac{1}{2})=\sqrt{\pi}$, which proves the assertion.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.**

Let V=the required volume ; A=the required area.

$$\therefore\ V=\int\int\int dxdydz=\int\int zdxdy=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}e^{-(x^2+y^2)}\,dxdy.$$

$$\therefore\ V=\left[\int_{-\infty}^{+\infty}e^{-x^2}dx\right]\left[\int_{-\infty}^{+\infty}e^{-y^2}dy\right].\quad A=\int\int dxdz=\int zdy=\int_{-\infty}^{+\infty}e^{-x^2}dx.$$

But $\int_{-\infty}^{+\infty}e^{-x^2}dx=\int_{-\infty}^{+\infty}e^{-y^2}dy$. $\quad\therefore V=\left[\int_{-\infty}^{+\infty}e^{-x^2}dx\right]^2=A^2$.

III. **Solution by C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts.**

$z=e^{-(x^2+y^2)}$. Applying formula for volume, $V=\int\int zdydx$, we have $V=4\int_0^\infty\int_0^\infty e^{-(n^2+y^2)}dydx$......(1). Also let $y=0$. Then $z=e^{-x^2}$ is the equation of section made by zx plane. Area$=2\int_0^\infty e^{-x^2}dx$........(2). Let this be equal to a......(3). Now put (1) in form of $V=4\int_0^\infty\int_0^\infty e^{-x^2}e^{-y^2}dydx$. Integrating with reference to y in accordance with (3), we have $V=2\int_0^\infty ae^{-x^2}dx=2a\int_0^\infty e^{-x^2}dx=a^2$, also in accordance with (3).

Professor William Hoover did not solve this problem but referred to Todhunter's Integral Calculus, Art. 204, where a good solution is given.

PROBLEMS.

49. **Proposed by B. F. BURLESON, Oneida Castle, New York.**

Find (1) in the leaf of the strophoid whose axis is a the axis of an inscribed leaf of the lemniscata, the node of the former coinciding with the crunode of the latter. Find (2) in a leaf of the lemniscata whose axis is b the axis a of an inscribed leaf of the strophoid, the node of the former also coinciding with the crunode of the latter.